1 Determinant Coefficient Matrix

Definition 1 (Superlogarithm Matrix)**.** Let:

$$S_{jk}(x) = (k\log(x))^{(j-1)} - \cdot k!\delta_{(j-1)k} \text{ if } 1 \leq (j,k) \tag{2}$$

$$S_{jk}(x)_{nm} = (S_{jk}(x))(1-\delta_{mk}) + \delta_{j1}\delta_{mk} \text{ if } 1 \leq (j,k) \leq n \tag{3}$$

define the matrices $\mathbf{S}(x)$ and $\mathbf{S}(x)_{nm}$, where δ_{jk} is Kronecker delta.

Definition 4 (Determinant Coefficient Matrix)**.** Let:

$$\det(\mathbf{S}(x)_{nk}) = \left(\prod_{i=0}^{n-1} i!\right) \sum_{j=0}^{\binom{n}{2}} C_{jk}(n)\log(x)^j \tag{5}$$

define the matrix $\mathbf{C}(n)$ for all $0 \leq k \leq n$.

Conjecture 6 (Partial Parameterization of $\mathbf{C}(n)$)**.**

$$C_{jk}(n) = \begin{cases} 1 & \textit{if } (j,k) = (0,0), \\ 1 & \textit{if } (j,k) = (0,n), \\ 0 & \textit{if } 0 \leq j \leq (k-1) \textit{ and } (j+1) \leq k \leq (n-1), \\ \frac{A_j}{j!} & \textit{if } 0 \leq j \leq (n-1) \textit{ and } k = n, \\ \frac{n^k}{k!} & \textit{if } (j-k) = 0, \\ \frac{A_{(j-k)}n^k + B_{jk}(n)}{(j-k)!k!} & \textit{if } 0 \leq (j-k) \leq 5, \\ (-1)^k\binom{n}{k}\frac{n}{k}(k-1) & \textit{if } j = (\binom{n}{2}-1) \textit{ and } 1 \leq k \leq n, \\ (-1)^{k+1}\binom{n}{k} & \textit{if } j = \binom{n}{2} \textit{ and } 1 \leq k \leq n, \\ 1 & \textit{if } (j,k) = (\binom{n}{2}, 0), \\ (\textit{unknown}) & \textit{otherwise.} \end{cases} \tag{7}$$

where $A_n = n!C_{nm}(m)$ *for all* $m > n$*, and:*

$$B_{jk}(n) = \sum_{i=0}^{j-k} (-1)^{\delta_{i0}} \binom{j-k}{i} \left((j-k-i)^i i^k - i^i (j-k-i)^k\right) n^{j-k-i} \tag{8}$$

Examples of A_m *are:*

$$\{A_n\}_{n=0}^{6} = \{1, -1, -4, -21, -160, -1505, -17136\} \tag{9}$$

$$\left\{\frac{A_n}{n!}\right\}_{n=0}^{6} = \{1, -1, -2, -\frac{7}{2}, -\frac{40}{3!}, -\frac{301}{4!}, -\frac{2856}{5!}\} \tag{10}$$