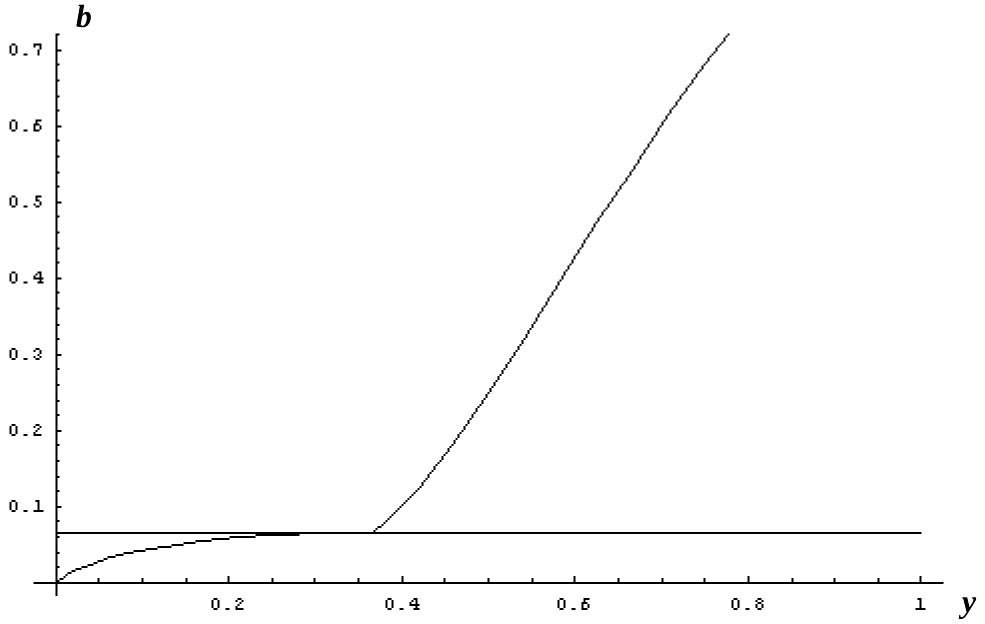
Fig. 1 – First branch of $b = e \wedge \dfrac{\text{plog}\left(y \cdot \ln y\right)}{y}$, for $\text{plog}\left(z\right) = W_0\left(z\right)$

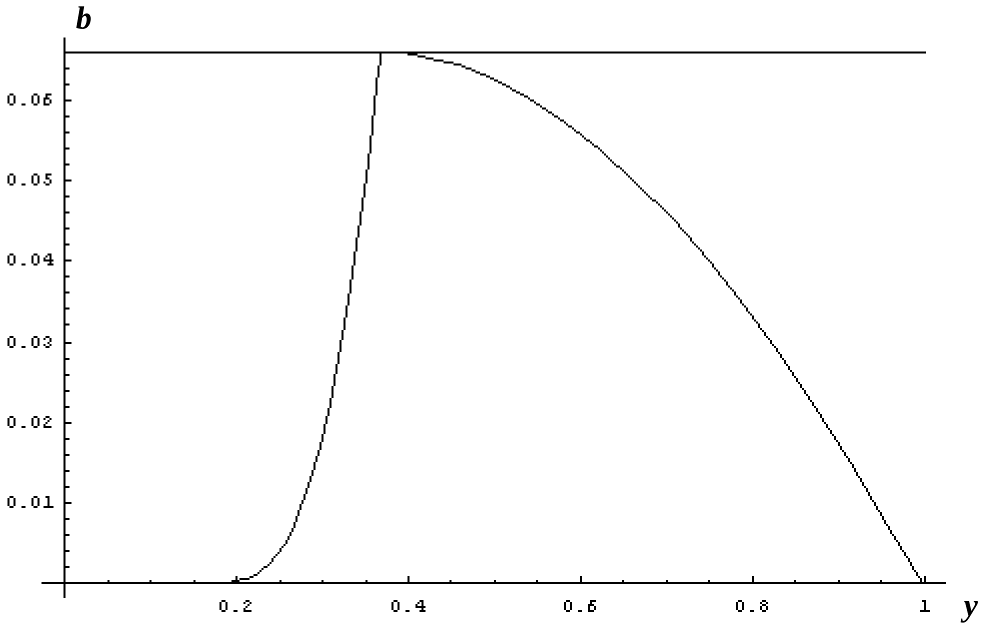


The figure shows the real values of the first branch of $b = b(y)$ and a tangent discontinuity at:

$$y = 1/e = 0.367879441..$$

with: $b = e^{-e} = 0.065988036..$

Fig. 2 – Second branch of $b = e \wedge \dfrac{\text{plog}\left(y \cdot \ln y\right)}{y}$, for $\text{plog}\left(z\right) = W_{-1}\left(z\right)$

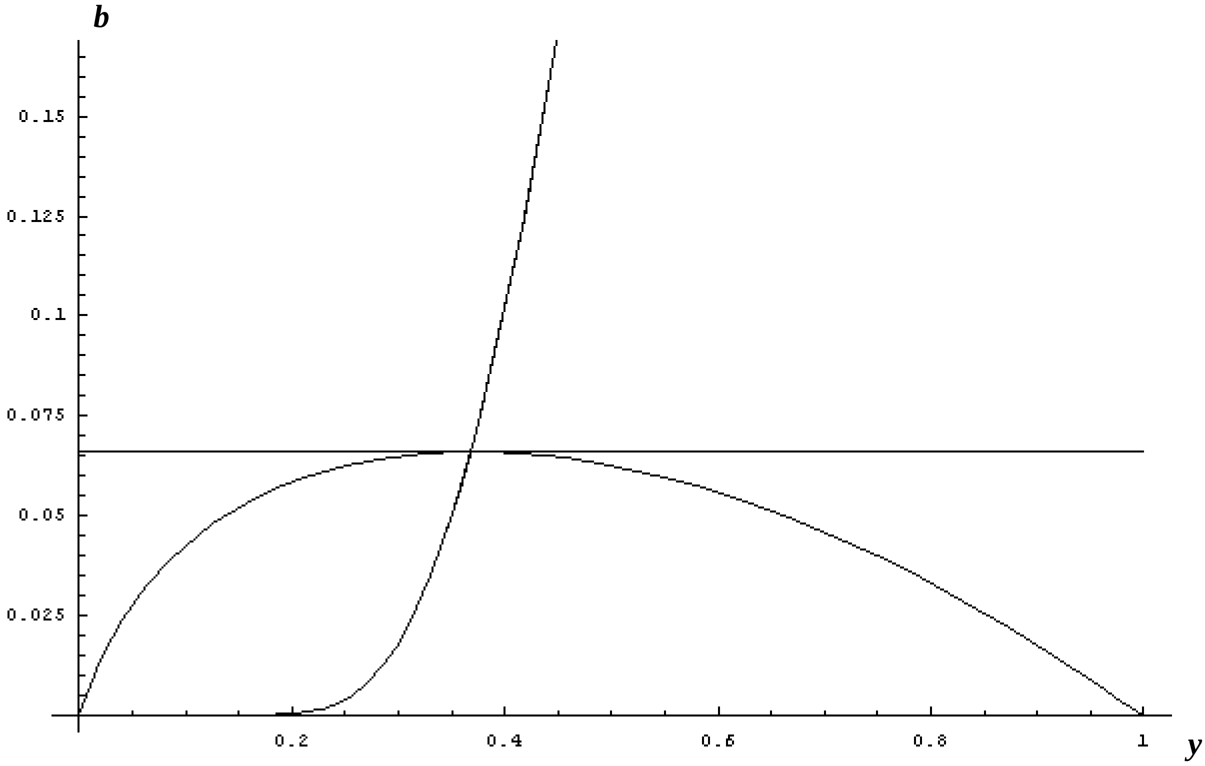


The figure shows the real values of the second branch of $b = b(y)$ and a tangent discontinuity at:

$$y = 1/e = 0.367879441..$$

with: $b = e^{-e} = 0.065988036..$

Fig. 3 – Joint plot of $b = e \wedge \dfrac{\text{plog}\left(y \cdot \ln y\right)}{y}$, for $\text{plog}\left(z\right) = \left\{W_{-1}\left(z\right), W_0\left(z\right)\right\}$

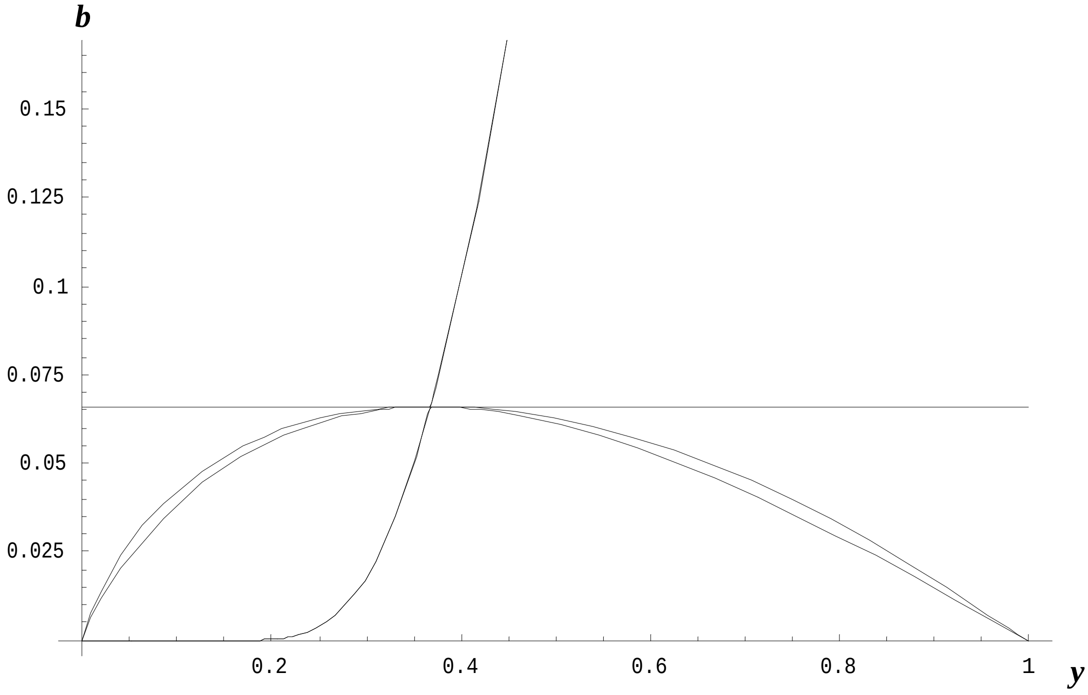


The plot shows the "middle value" of $b = b(y)$, also described by the selfroot $b = \sqrt[y]{y} = y^{1/y}$ and the full perimeter of the zone, showing a maximum at:

$$y = 1/e = 0.367879441..$$

with: $b = e^{-e} = 0.065988036...$

Fig. 4 – Comparison between an "old" approximation and the new proposed formula



Old formula (lower perimeter, insufficient approximation)

$$b = \kappa \cdot (y^{-y} - 1), \qquad \text{with } \kappa = \frac{e^{-e}}{e^{1/e} - 1} = \frac{\beta}{\eta - 1}$$

New formula (upper perimeter, correct values, I hope ...):

$$y = b^y$$
$$y = b^{b^y}$$
$$\log_b y = b^y$$
$$\log_e y = b^y \cdot \log_e b = e^{y \cdot \log_e b} \cdot \log_e b$$
$$y \cdot \log_e y = y \cdot \log_e b \cdot e^{y \cdot \log_e b} = \log_e y^y$$
$$y \cdot \log_e b = \operatorname{plog}\left(\log_e\left(y^y\right)\right)$$
$$\boxed{b = e \wedge \frac{\operatorname{plog}\left(\log_e\left(y^y\right)\right)}{y}}$$

$$b = e \wedge \frac{\operatorname{plog}\left(\log_e\left(y^y\right)\right)}{y} = \exp\left[\frac{\operatorname{plog}\left(-\log_e\left(y^{-y}\right)\right)}{y}\right] = \exp\left[\log_e y \cdot \frac{\operatorname{plog}\left(-\log_e\left(y^{-y}\right)\right)}{\left(-\log_e\left(y^{-y}\right)\right)}\right]$$

$$b = y \wedge \frac{\operatorname{plog}\left(-\log_e\left(y^{-y}\right)\right)}{\left(-\log_e\left(y^{-y}\right)\right)} = y \wedge {}^{\infty}\left[y^{-y}\right] = y \wedge {}^{\infty}\left[\frac{1}{y^y}\right]$$

NB : This formula "includes" the values obtained by $b = y \wedge \frac{1}{y}$ (the selfroot). **Right?**

Fig. 5 – Plot of $y = (0.025)^x$ and $y = \log_{0.025}(x)$, odd iterates (r = -1 and r = 1)

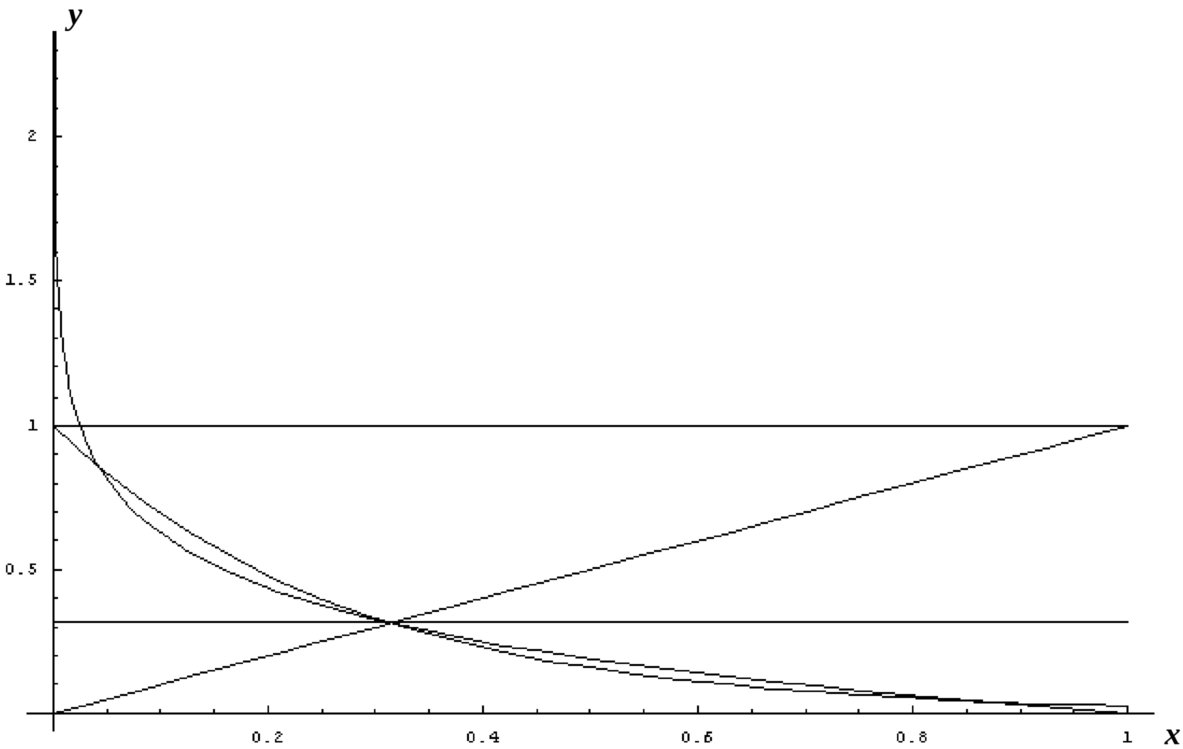


Fig. 5 shows three crosspoints between the exp and the log diagram, one of which is a fixpoint., where log *x* = exp *x* = *x*. The base is $b = 0.025 < e^{-e}$.

<u>Fig. 6</u> – Plot of $y = (0.025)^{(0.025)^x}$, $y = \log_{0.025}(\log_{0.025}(x))$, even tet. ($r$ = -2, r = 0, r = 2)

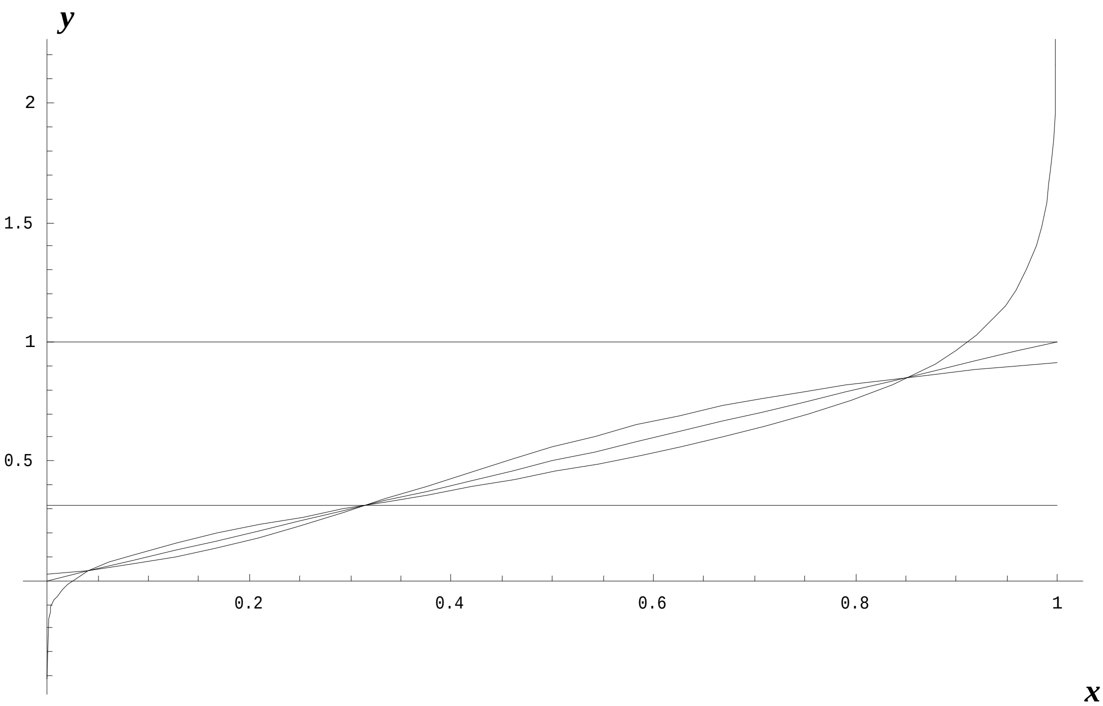
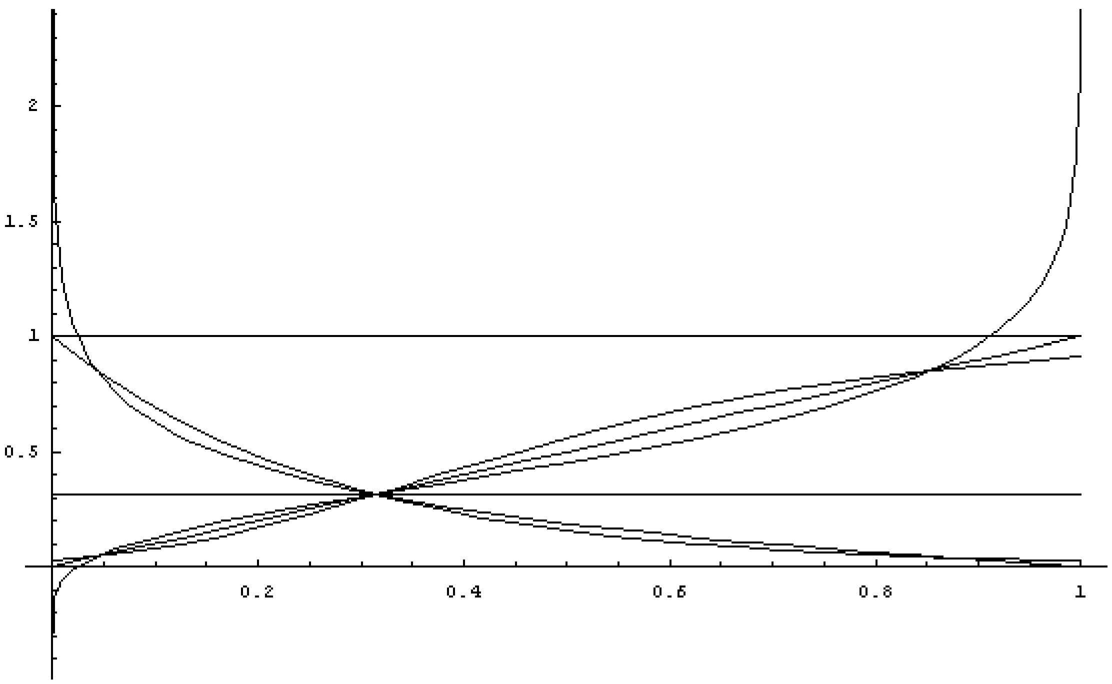
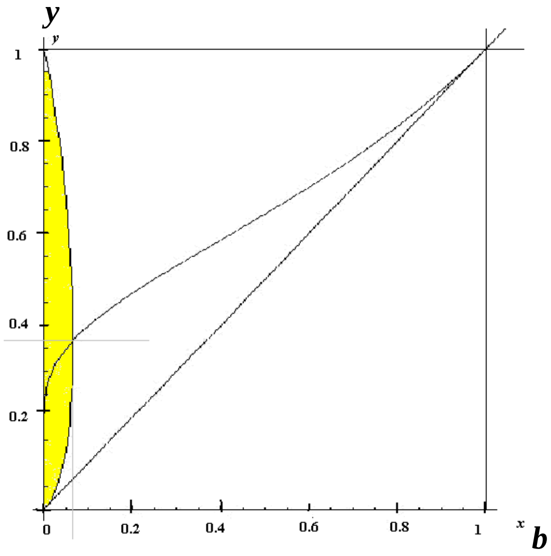


Fig. 6 shows three crosspoints between the loglog and the expexp diagrams, all of which are fixpoints, where loglog(*x*) = expexp(*x*) = *x* (base b = 0.025)

<u>Fig. 7</u> – Superposition of the plots of Fig. 4 and Fig. 5 (base b = 0.025).



<u>Fig. 8</u> – Qualitative inversion of Fig. 3.



The bifurcation point is shown at $b = e^{-e} = 0.065988036..$, where $y = 1/e = 0.367879441..$
The plot shows three values for *y*, at $0 < b < e^{-e}$

NB. We need an analytical inversion of $b = e^{\wedge}\dfrac{\text{plog}\left(\log_e\left(y^y\right)\right)}{y}$:

I don't know how to proceed.

GFR – 2008-05-01 (May Day!)