New thinking about math infinity
by Alister “Mike Smith” Wilson

(My understanding about some historical ideas in math infinity and my contributions to the subject)
For basic hyperoperation awareness, try to work out 3^^3, 3^^^3, 3^^^^3 to get some intuition about the patterns I’ll be discussing below. Also, if you understand Graham’s number construction that can help as well. However, this paper is mostly philosophical.
So far as I am aware I am the first to define Nopt structures.
Maybe there are several reasons for this: (1) Recursive structures can be defined by computer programs, functional powers and related fast-growing hierarchies, recurrence relations and transfinite ordinal numbers. (2) There has up to now, been no call for a geometric representation of numbers related to the Ackermann numbers. The idea of Minimal Symbolic Notation and using MSN as a sequential abstract data type, each term derived from previous terms is a new idea.
Summarising my work, I can outline some of the new ideas: (1) Mixed hyperoperation numbers form interesting pattern numbers. (2) I described a new method (butdj) for coloring Catalan number trees the butdj coloring method has standard tree-representation and an original block-diagram visualisation method. (3) I gave two, original, complicated formulae for the first couple of non-trivial terms of the well-known standard FGH (fast-growing hierarchy). (4) I gave a new method (CSD) for representing these kinds of complicated formulae and clarified some technical difficulties with the standard FGH with the help of CSD notation. (5) I discovered and described a “substitution paradox” that occurs in natural examples from the FGH, and an appropriate resolution to the paradox. This substitution paradox is well-known in computer science (object oriented programming) but not so well-known in mathematics, although related to the theory of types. (6) I described the original concept of minimal symbolic notation and invented the seed-theta notation for notating hyperoperations. (7) I gave an original geometric representation of base(m) Ackermann function using seed-theta nopt structures with CSD visualisation technique, and noted an important exponential property of this geometry. (8) I generalised the recursion patterns from nested exponential power towers into NOPT form and clarified some technical issues concerning correct definition of the ordertype of a NOPT structure. (9) I accurately described a meaningful number realm (called naropt structures) where the famous Graham’s number resides. This realm relates to a specific range of Conway numbers or Bowers’ operators. (10) The well-known repdigit (repeated digit) SPN numbers can be extended into the NOPT hierarchy. This means that, in theory, SPN numbers can be extended in standard binary tick-tock fashion into the NOPT hierarchy. In reality, it is only relatives of repdigit numbers with low information content that can be described in this way. It’s not possible to see how to do this using the standard well-known fast-growing hierarchy. This has ramifications for set theorists in terms of what has been described by various authors as “right-sizing the infinite” or in other words, figuring out how to talk better about infinite ordinals such as omega and epsilon-zero. Even the low-information repdigit SPN numbers can’t be described beyond the hyperoperation realm into naropt structures (where Graham’s number resides). This means we can’t even be sure where the first infinite ordinal number, omega resides, which contradicts the standard set-theoretical wisdom that says that omega is well-defined as the first infinite number greater than all the finite numbers. (11) This has the amazing corollary that numbers described (by finite methods) either beyond g2 or beyond Bowers’ {{{1}}} operator are effectively infinite numbers. Infinite phenomena can be observed in some finite mathematical objects and finite phenomena can be observed in some infinite mathematical objects.
(12) I described the curious looking slow-growing nopt structure. (13) I described a technique of using binary filter codes to define sequences that are projections through the ordinal hierarchy similar to transitional sequences through the hyperoperator hierarchy. (14) There are interesting transitional sequences from exponentiation to tetration and also through the hyperoperator hierarchy. (15) There are 8 folding methods for nopt structures (LU, RU, LD, RD, UL, UR, DL and DR)

(16) I gave some reasons for choosing the "canonical method" as fold-LU so that the "answer value" is the first thing you observe (in the top-left-corner position of the structure) when reading some text from right-left and top-down. Although the 8 fold-URDL operations are well known from computer science they haven't until now been used for giving a geometric representation of numbers related to the Ackermann numbers (so that computational pathways and ordertype phenomena can be pictured and described). (17) Nopt structures encapsulate the requirements of a minimal notation whether via formula using an equation editor etc or via colored-square diagrams. (18) By analogy to nepts (via the definition of nopts) we can describe naropts, or nested Knuth arrow towers. A particular subsequence from the Conway numbers corresponds with naropts.

The Theory of Large numbers

A lot of mathematicians don't believe there is a theory of large numbers. Naturally, I and many of the more serious contributors to the xkcd forum would disagree. The MSC2010 subject classification doesn't have a classification for this theory, Wolfram Mathworld doesn't mention the Fast Growing Hierarchy (FGH), the few researchers in tetration have asked for tetration results to be considered as relevant for this encyclopedia. Clifford Pickover's recently published Math Book is excellent but it doesn't mention Andrzej Grzegorczyk and the FGH, although he does mention Ronald Graham in the last entry of the book. I suspect that an entry about FGH was considered but then rejected due to likelihood of making the book slightly less accessible. In other words, the subject is treated as something of a messy trade secret in maths. I think that the 21st century should be more accepting that this is teachable material at the university level. When I was a student there wasn't a course available in number theory, let alone anything about the narrower subject of subrecursive hierarchies. And yet the material is inspirational and resolves a latent natural curiosity that maths students have about the subject. That is just personal opinion, but considering how much finite and infinite are common and obviously useful concepts in maths, I believe clear thinking about the subject is useful. I only read about Graham's number when I was a public servant and my boss lent me the well-known Puffin book "Curious and interesting maths" by David Wells. When I read the book it greatly surprised me that Ronald Graham was able to describe such an incomprehensible number that passed the usual mathematical criteria for being well-defined. Actually some mathematicians doubt the "reality" of Graham's number presumably either because they consider the number as effectively infinite or they regard the number as poorly defined due to the "ambitious construction" of recursing on Knuth arrows. Well, my research has clarified a lot about the nature of the hyperoperator hierarchy and how it is built up from previous stages. Part of my research is about the important region of numbers from the hyperoperator hierarchy and part of it is about the esoteric number environment of Graham's number. Some people with non-math backgrounds have been able to appreciate my work, so I really hope the maths professors can gradually accept the results from my research for they are accurate results given acceptance of some stated caveats concerning the original technique. I am guessing that mathematicians are probably somewhat bemused and intrigued about the ideas I present in the papers.

Combinatorial brainstorming is the art of using microscopic detailed reasoning and lateral thinking to uncover phenomena in established subjects that have been hidden over time. Whilst the academic tradition is rooted in the tradition of building on other people's work via the tradition of scientific citations there are some researchers with wide-ranging but superficial knowledge that have applied the collected gems of understanding to a specific area in order to make a deep observation about the specific subject area. It is probable that all the great thinkers such as Gauss and Euler used combinatorial brainstorming techniques more than others in order to latch onto and develop their ideas. Patrick Gunkel's ideonomy is probably a close formulation of these ideas and his talents are recognised by MIT. Some of his combinations of ideas are a little strange, but then again, this goes with the territory of exploration of new concepts via brainstorming technique. In other words, there is a lack of refinement and coherence in some of the exemplars, but as a demonstration of combinatorial brainstorming it's a very useful contribution. Wolfram's computational knowledge historical timeline poster indirectly refers to the technique of combinatorial brainstorming as well. The techniques can be applied in maths as well to varying degrees of success. Alan Turing is important for suggesting a flexible general purpose computer could be developed, together with his forward thinking concerning Artificial Intelligence and cryptography. Andrzej Grzegorczyk is also important in the history of computer science because he carefully and successfully defined the Fast Growing Hierarchy and related Fast Iteration hierarchies and Grzegorczyk Hierarchy. Wikipedia allows this information to be shared, and public education benefits for publishing this information. Graham's number is very well known and gets a mention in Clifford Pickover's classic "Math Book" but the FGH is not mentioned in his book, probably due to its complexity and potential inclusion not enhancing the sales popularity of the book. In my papers I have made a serious attempt to integrate knowledge about Graham's number, the FGH and a *new* structure I defined called Nopt structures (and the related geometry of noptiles). My papers can be read by people from the Foundations Of Math (FOM) community, but the problem is that a new theory may interfere with another paradigmatic theory. The serious mathematicians on Eretrandre website do an great job concerning developing the theory of tetration that is one of the early 21st century hot areas of mathematical research. Universities seem to be slow about picking up and using these new ideas due to conservative traditions. My papers are completely original and clarify the issues mentioned above. Meanwhile in the FOM community, Giuseppe Peano is congratulated for his Peano axioms and the principle of mathematical induction while it is clear that his more significant contribution was realising the importance of space-filling curves, that was continued by David Hilbert and others and more recently by Bill Gosper and myself. Also, Georg Cantor is congratulated for his diagonalisation argument, whereas we can also observe the accurate principle of multi-layered nested limited recursions. For some reason, it seems Cantor didn't discuss hereditary base in the finite numbers, although it clearly was the inspiration for his theory of transfinite ordinals, and he didn't mention the Catalan numbers when he considered clustered forms of recursion and Goodstein used hereditary base by incorporating the concept into his Goodstein theorem. And Conway's surprising and far-reaching recurrence relation behind the Conway chained arrow notations is an important idea but difficult to concentrate on and understand about. Andrew Snowden's height density problem is not very well-known, even though it derives from a generalisation of the Fundamental Theorem of Arithmetic, maybe because incorporating hereditary base into the Fundamental Theorem of Arithmetic is often regarded as an unnecessary complication. Meanwhile, there are distributed computing projects trying to find largest primes in the different prime number classes. In the 21st century, university math curricula advisors should consider a stage of pedagogical consolidation following some ideonomic principles and thoughtful memetic explorations of the great online encyclopedias of our time in order to carefully survey the variety of ideas in number theory and other branches of maths. We are living in the fortunate times of living in the information age where anyone with a computer can view epic historical math videos such as the OEIS video and David Metzler's explanations of the FGH and Goodstein sequence. Historically, subrecursive hierarchies have always been the most troublesome thing to communicate in mathematics. In 1908, Oswald Veblen wrote his famous paper about transfinite ordinals. Recently, Paul Budnik has contributed his ordinal calculator. Grzegorczyk's idea about the Fast Growing Hierarchy and Fast Iteration Hierarchies was important being a natural idea but with convoluted implications, concerning emerging complexity. I think it's good for math lecturers to be honest about this complexity with math students at some level of the curricula, because it is humbling about the limits of knowledge, and supports the thinking about how simple systems produce complex behaviour.

There is an artistic side to my math ideas, that makes it a curiosity in the world of maths. What's different about my work is that the art-side is an essential part of the maths, there's no way the math could be properly and effectively communicated via formula type setting software. The Color Square Diagrams is the logical way to understand these kinds of numbers and functions. The CSD technique is versatile and can illustrate various ideas in number theory and combinatorics using the concept of “small world examples for presentation, practice and organisation” and is a technique commonly used in the Demonstrations Projects from the Mathematica website.

The 19 math animations listed in the paper Composite Mulanept Patterns function on two levels:
(1) as noptiles (non-standard geometric tiling patterns and associated space-filling curve) and
(2) as Nopt structures (where the various functional interpretations are added).
They can help to understand the basics about the theory of computation. The Nopt structures require many animations to be explained clearly, for example (1) the 8 folding patterns for the pure noptiles, (2) a natural and intuitive computational pathway and (3) the natural, practical and visually smooth transitional sequences.

In the school system we learn addition, multiplication, exponentiation and the natural numbers.
At secondary school we learn about the exponent laws. In university math curricula there’s little said about the patterns, character or properties of big numbers except in the subjects of set theory, computability theory, computational complexity theory and the theory of computation. These are difficult graduate level courses to understand. There’s also number theory, which is another difficult subject but only relatively smaller numbers can be analysed using the methods and techniques of number theory. In set theory, there are the transfinite ordinals and cardinals, the continuum hypothesis (CH) the subject developed by Georg Cantor in the 19th century, and regarded as the first problem from David Hilbert’s famous list of 23 unsolved problems. The CH and GCH (generalised CH) are still considered by set theorists, but the results of Kurt Godel and then Paul Cohen (1963) seem to leave the foundations of maths in an uncertain situation, not that that matters to a lot of set theorists who continue researching undeterred. Some other famous mathematicians are:
Andrzej Grzegorczyk (a Polish logician, Fast growing hierarchy)
Wilhelm Ackermann (defined the famous Ackermann number sequence)
Donald Knuth (wrote important books about the art of computer programming)
John Horton Conway (an accomplished and prolific mathematician)
Ronald Graham (Ramsey theory and the famous Graham’s number)
David Madore (transfinite ordinals represented by trees)
Paul Budnik (came up with a transfinite ordinal calculator)

And some good and useful websites include:
Websites about large “finite” numbers
www.iteror.org
esoteric, obscure, colourful and difficult to follow and understand
www.polytope.net/hedrondude/home.htm
Bowers came up with many colourful names for various very big numbers, and has done amazing work with polyhedra and polychora.
www.mrob.com
Munafo covers various topics in big numbers, number theory and computer science.